A summary of the more recent papers on the completeness of the Logic of Bunched Implications

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The history of attempts to give completeness results for the Logic of Bunched Implications (BI) is quite complex. The book *The Semantics and Proof Theory of the Logic of Bunched Implications* (D. Pym, 2002, Springer, with some errata at https://extras.springer.com/?query=978-1-4020-0745-3) contains early rough notes with many errors and omissions of detail. It should not be regarded as a primary source for BI.

The following, in reverse chronological order, are the main recent sources:

1. Alexander Gheorghiu, Tao Gu, and David Pym. Proof-theoretic Semantics for the Logic of Bunched Implications. *Studia Logica*, 2025, doi.org/10.1007/s11225-025-10202-z.

   This is perhaps the most natural complete semantics for BI. It employs 'base-extension semantics'.

2. Alexander Gheorghiu and David Pym. Semantical Analysis of the Logic of Bunched Implications. *Studia Logica*, 2023, doi.org/10.1007/s11225-022-10028-z.

   This is a quite general treatment of completeness, based on ternary relations and employing a novel proof technique. It also includes an extensive discussion of the issues around completeness and its failures.

3. Several papers by Docherty and Pym explore similar issues for logics in the BI family and obtain useful completeness theorems:

   (a) Simon Docherty and David Pym. Modular Tableaux Calculi for Separation Theories. In: *Proc. FoSSaCS 2018*, LNCS 10803, Springer, 2018

   (b) Simon Docherty and David Pym. Intuitionistic Layered Graph Logics: Semantics and Proof Theory. *Logical Methods in Computer Science*, https://lmcs.episciences.org/4942.

4. D. Galmiche, D. Méry, and D. Pym. The Semantics of BI and Resource Tableaux. *Math. Struct. Comp. Sci.* (2005) 15, 1033–1088.

This paper includes a wide range of completeness results, including the monoid-based Beth-style semantics (BRM) and the relational semantics, but it contains some errors and points of confusion. The results for the PDM and KRM semantics are correct for the tableaux system TBI. In some places BI is written for TBI, possibly implying other results (e.g., for LBI) that are not obtained. In the result for BRM, the join-rules of the tableaux calculus were inadvertently omitted, but the result holds in their presence. The claim in the abstract that decidability is obtained is not substantiated; rather, a possible approach through the FMP is suggested.

5. D.J. Pym, P.W. O'Hearn, and H. Yang. Possible worlds and resources: the semantics of BI. *Theoretical Computer Science* 315(1): 257–305, 2004.

   This paper includes the completeness result for the Grothendieck topological version of the monoidal semantics, which is not the elementary monoidal semantics.